Large Planar Array Thinning with an Improved Multi-Objective Genetic Algorithm

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An improved multi-objective optimizer based on the nondominated sorting genetic algorithm II (NSGA-II) is presented for large planar array thinning in this work. The iterative fast Fourier transform (IFFT) technique with a judge factor is introduced into the optimizer to accelerate the convergence. In the early phase of the optimization algorithm, the global characteristics of GA occupy a major position and the powerful local characteristics of IFFT in the late phase. Thus, this proposed algorithm can not only effectively avoid being trapped into the local optimum, but also possess a fast convergence for large array thinning. A representative example shows the good performance of the proposed algorithm.

***Index Terms*—Array thinning, nondominated sorting genetic algorithm II (NSGA-II), iterative fast Fourier transform (IFFT).**

I. Introduction

Since the first studies were carried out in the 1960s [1], [2], thinned arrays have been an object of intense research due to several advantages associated to their lower cost, weight, power and complexity than fully filling arrays. Moreover, the improvement of the sidelobe level (SLL) of the thinned arrays, especially for large ones, is attracting much attention.

Initially, the statistical method was used to select the element locations to provide the desired density taper [1]. In recent years, the optimization methods, such as genetic algorithms (GAs) [3] and particle swarm optimizer (PSO) [4], have been used for thinned array design. However, these methods fall far short of optimum configurations for large arrays due to the very intensive computational burden. An iterative fast Fourier transform (IFFT) technique is proposed to arrive at thinned arrays [5]. IFFT can realize array thinning very fast, and what is more, it can get thinned element distributions for both linear as well as planar arrays with more than 1000 elements. However, this technique is easy to be trapped into the local optimum. Therefore, a sufficient number of trials are applied so that it can find the thinned element distribution which represents the global optimum solution.

In this work, an improved multi-objective optimization algorithm based on nondominated sorting genetic algorithm II (NSGA-II) [6], [7] is introduced to meet the large planar array thinning requirements of a given filling factor and the lowest possible SLL. Combined with IFFT which performs only once instead of multiple times in each generation of GA, this improved algorithm takes on a global search ability on one hand and good local convergence on the other. In addition, a judge factor applied to decide whether IFFT is executed, guarantees the global characteristics of the algorithm in the early phase and the powerful local characteristics in the late phase. A numerical example of large planar array thinning is provided and the obtained array pattern shows a low SLL with a given array filling factor.

II. Basic Theory of Thinned Array

According to the classical principle of pattern multiplication, the far-field F in the direction (θ, φ) can be written as

$$F(u,v) = EF(u,v)AF(u,v) \tag{1}$$

where EF is the embedded element pattern, AF is the array factor, and u and v are defined as $u = \sin\theta\cos\varphi$ and $v = \cos\theta\cos\varphi$, respectively. If a planar array consists of N rows and M columns of elements with an element spacing d_x in the x-direction and d_y in the y-direction, $AF(u, v)$ can be characterized as

$$AF(u,v) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} A_{mn} e^{jk(md_x u + nd_y v)} \tag{2}$$

where A_{mn} is the complex excitation. The array factor is related to the element excitations through an inverse Fourier transform, which means that the element excitations can be derived from the array factor through a direct Fourier transform, and then the far-field F can be obtained.

In terms of thinned arrays, A_{mn} of the removed elements (turned off) is zero, and the number of radiating elements depends on the array filling factor f, which is defined as the ratio of fraction of the radiating elements in relation to the total number of the corresponding full array.

III. Improved Multi-Objective Genetic Algorithm

In the array thinning process, the element number and the peak sidelobe (PSL) are the two optimization objectives. The objective functions can be written as

$$\begin{cases} Objv_1 = \min | N_{\mathrm{E}} - N_{\mathrm{Full}} \times f | \\ Objv_2 = \min(\mathrm{PSL}) \end{cases} \tag{3}$$

where N_{E} is the element number of current individuals and N_{Full} is the total element number of the corresponding full array, f is the filling factor of the expected thinned array.

The flowchart of the proposed algorithm is shown in Fig. 1. In NSGA-II, a random initial population with N_{E} individuals is generated and the fitness values of individuals are calculated.

Then nondominated sorting and crowding-distance calculation are given. A new mixed population is constituted by three parts, namely, the offspring population obtained from GA operators, the offspring one achieved with IFFT and the parent one. At last, a trimming operation is applied to the mixed population after the fitness values are evaluated.

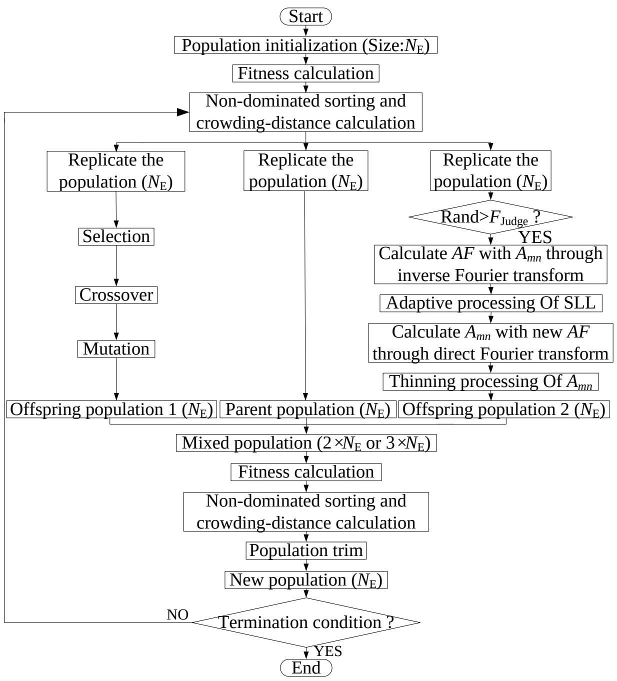

Fig.1. Flowchart of the improved multi-objective genetic algorithm.

Different from the IFFT technique in [5], here the iteration operation runs only once for the inhibition of prematurity in each generation of GA. What is more, the IFFT technique applied to the current generation depends on whether a random number (*Rand*) drawn from [-1, 1] is larger than an introduced judge factor F_{Judge}. The F_{Judge} is defined as

$$F_{\text{Judge}}^{t+1} = \alpha \times F_{\text{Judge}}^{t} \quad (4)$$

where α is a constant similar to the cooling factor of a cooling schedule in the simulated annealing [8], and t and t+1 denote the tth and (t+1)th generations, respectively. For simplicity, F_{Judge}^{0} can be selected as 1 and α be 0.9 in this work. In addition, a lower limit of F_{Judge} is given in order to guarantee that the algorithm possesses the quantitative global characteristics in the late phase.

When the improved algorithm is applied to the array thinning, a set of nondominated solutions, *viz.*, the element number and the PSL constitute the *pareto front*. Only the solution in which the $Objv_1$ equals to zero and the $Objv_2$ is the minimum one is chosen as the best solution in current generation. After a few generations, the global best solution for the thinned array with a given filling factor can be found.

IV. Numerical Example

The presented thinning algorithm is tested on a circular planar array with a diameter of 100 wavelengths and a filling factor of 30%. The considered array features an embedded isotropic element pattern and the element spacing is selected as half wavelength. The algorithm uses 50 agents for 200 iterations, and the lower limit of F_{Judge} is chosen as 0.4.

Fig. 2 shows the element distribution and the u-cut of the far-field of the thinned array. It can be observed that the PSL is -35.9 dB which is 0.4 dB lower than the result in [5]. The best thinned element distribution is found after only 35 generations. 20 trials are applied to the thinning algorithm and almost all obtained PSLs are the same. In addition, the traditional optimization methods are not able to thin an array with such a large size.

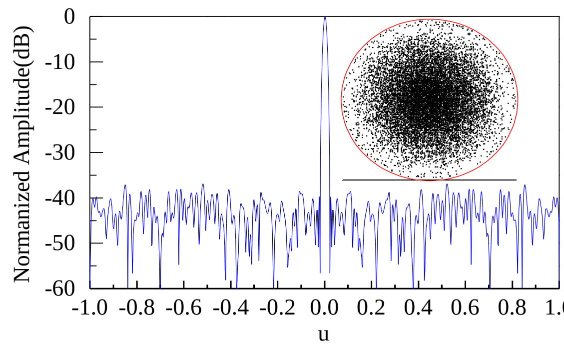

Fig. 2. Best element distribution and u-cut of the far-filed of the thinned array.

V. Conclusion

This work introduces an improved multi-objective genetic algorithm in large planar array thinning. Combined with the IFFT technique, the GA optimizer possesses a powerful global optimization capability and a fast convergence speed. A numerical example shows that the proposed algorithm can achieve the low sidelobe results in large array thinning.

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